Year 1 – Addition and Subtraction

Key learning:

- Solve problems involving counting, adding and subtracting in the context of numbers, measures or money, for example to 'pay' and 'give change'.
- Describe ways of solving puzzles and problems, explaining choices and decisions orally or using pictures.
- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.
- Represent and use number bonds and related subtraction facts within 20.
- Add and subtract one digit and two digit numbers to 20, including zero (using concrete objects and pictorial representations). Relate addition to counting on; recognize that addition can be done in any order; use practical and informal written methods to support, understand subtraction as take away, find a difference by counting up; use practical and informal methods to support the subtraction.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9.

Key vocabulary:

+, add, more, plus
number line
make, sum, total
altogether
score
double, near double
one more, two more….ten more
count on
how many more to make…?
How many more is…than….?
How much more is….?

-. Subtract, take (away), minus
Leave
Count back
How many are left/left over?
How many have gone?
One less, two less, ten less…
How many fewer is….than….?
How much less is…?
Difference between
Half, halve

=, equals, sign, is the same as

Mental calculations:

- Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number.
- Count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens.
- Given a number, identify one more and one less.
- Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least.
- Read and write numbers from 1 to 20 in numerals and words.

Year 1 – Addition and Subtraction

Addition – Add with numbers up to 20.

Children should:

- Have access to a wide range of counting equipment, everyday objects and number lines and be shown numbers in different contexts.
- Read and write the addition (+) and equals (=) signs within number sentences.
- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them:

8 + 3 = ? 5 + 4 = ? 5 + 3 + 1 = ? ? + ? = 6

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

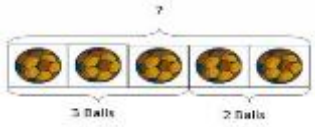
The four advanced counting strategies –

1. Counting on eg. 8 + 4 = ?
2. Counting up to eg. 8 + ? = 12
3. Counting back from eg. 12 – 4 = ?
4. Counting back to eg. 12 - ? = 8

Use numbered number lines to add, by counting on in ones.

Encourage children to start with the larger number and count on.

The part-part-whole model:



Use pictures to add two numbers together as a group or in a bar.

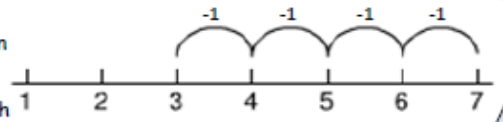
Subtraction – Subtract from numbers up to 20.

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Read, write and interpret number sentences with - and = signs.

Subtract by taking away

Count back in ones on a numbered number line to take away, with numbers up to 20:

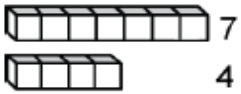


7 – 4 = 3

Model subtraction using hundred squares and numbered number lines/tracks and practically.

Find the 'distance between'

This will be introduced practically with the language '**find the distance between**' and '**how many more?**' in a range of familiar contexts.



'Seven is 3 more than four'

'I am 2 years older than my sister'

Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Year 2 – Addition and Subtraction

Key learning:

- Present solutions to puzzles and problems in an organised way; explain decisions, methods and results in pictorial, spoken or written form, using mathematical language and number sentences.
- Solve problems involving addition and subtraction, multiplication in contexts of numbers, measures or pounds and pence.
- Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem.
- Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting).
- Select a mental strategy appropriate for the numbers involved in the calculation.
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.
- Understand subtraction as take away and difference (how many more, how many less/fewer).
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Recall and use number bonds for multiples of 5 totaling 60 (to support telling time to nearest 5 minutes).
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
    1. a two-digit number and ones (units)
    2. a two-digit number and tens
    3. two two-digit numbers
    4. adding three one-digit numbers

Key vocabulary:

+, add, addition, plus, more, make, total, altogether, score
double
near double
one more, two more, ten more….one hundred more…
how many more to make..?
how many more is…than…?
How much more is..?

-. Subtract, take (away), minus, subtraction
leave
how many are left/over?
One less, two less, ten less…one hundred less…
How many fewer is…than…?
How much less is…?
Difference between..
Half/halve

=, equals, sign, is the same as
tens boundary

Mental calculations:

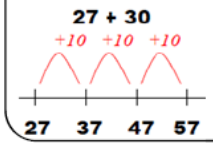
- Count in steps of 2, 3, and 5 from 0, and in 10s from any number, forward and backward.
- Find 1 or 10 more or less than a given number.

Year 2 – Addition and Subtraction

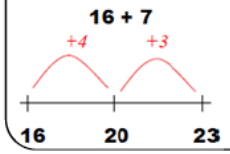
Addition – Add with 2 digit numbers.

Developing mental fluency with addition and place value involving 2 digit numbers, then establish more formal methods.

Add 2-digit numbers and tens:

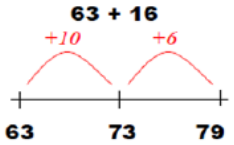


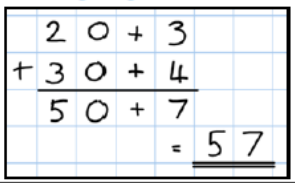
Add 2-digit numbers and units:



Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

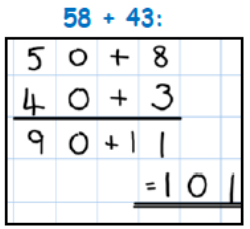
Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units:





STEP 1: Only provide examples that do NOT cross the tens boundary until they are secure with the method itself.

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 58 + 43).



STEP 3: Children who are confident and accurate with this stage should move onto the expanded addition methods with 2 and 3-digit numbers (see Y3).

To support understanding, pupils may physically make and carry out the calculation, for example using place value counters, then compare their practical version to the written form, to help them to maintain an understanding of it.

Subtraction – Subtract with 2 digit numbers.

Subtract on a number line by counting back, aiming to develop mental subtraction skills.

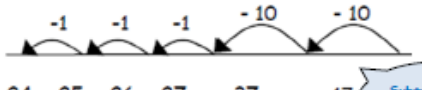
Use Dienes blocks for subtraction calculations too.

This strategy will be used for:

- **2-digit numbers subtract units** (by taking away / counting back) e.g. 36—7
- **2-digit numbers subtract tens** (by taking away / counting back) e.g. 48—30
- **Subtracting pairs of 2-digit numbers** (see below:)

Subtracting pairs of 2-digit numbers on a number line:

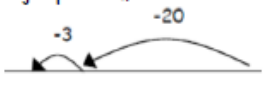
47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:



Then subtract units.

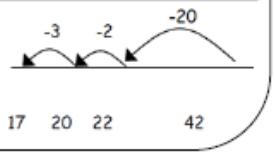
Subtract tens first.

Move towards more efficient jumps back, as below:



Combine methods with use of a hundred square to reinforce understanding of number value and order.

Teaching children to **bridge through ten** can help them to become more efficient, for example 42—25:



Year 3 – Addition and Subtraction

Key learning:

- Recall/use addition/subtraction facts for 100 (multiples of 5 and 10).
- Derive and use addition and subtraction facts for 100.
- Derive and use addition and subtraction facts for multiples of 100 totaling 1000.
- Add and subtract numbers mentally, including:
  - a three-digit number and ones (units)
  - a three-digit number and tens
  - a three-digit number and hundreds.
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.
- Estimate the answer to a calculation and use inverse operations to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Key vocabulary:

+, add, addition, more, plus
make, sum, total
altogether
score
double, near double
one more, two more... ten more... one hundred more
how many more to make...?
how many more is... than...?
how much more is...?

-, subtract, subtraction, take (away), minus
leave, how many are left/left over?
one less, two less... ten less... one hundred less
how many fewer is... than...?
how much less is...?
difference between
half, halve

=, equals, sign, is the same as
tens boundary, hundreds boundary

Mental calculations:

- Count from 0 in multiples of 4, 8, 50 and 100.
- Find 1, 10 or 100 more or less than a given number.
- Count up and down in tenths.

Year 3 – Addition and Subtraction

Addition – Add numbers with up to 3 digits.

Introduce the **expanded column addition** method:

```
  236
+  73
   9
  100
  200
  309
```

Add the **units** first, in preparation for the compact method.

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.

Move to the compact **column addition** method, with 'carrying':

Add **units** first.

'Carry' numbers **underneath** the bottom line.

```
 236
+ 73
 309
  1
```

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to 'carrying' for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

Remind pupils the actual value is '**three tens** add **seven tens**', not 'three add seven', which equals **ten** tens.

Subtraction – Subtracting with 2 and 3 digit numbers.

Introduce **partitioned column subtraction** method.

STEP 1: introduce this method with examples where **no exchanging** is required.

89 - 35 = 54

80 + 9
- 30 + 5
50 + 4

When learning to 'exchange', explore 'partitioning in different ways' so that pupils understand that when you exchange, the **VALUE** is the same ie **72** = 70+2 = 60+12 = 50+22 etc. Emphasise that the **value hasn't changed**, we have just partitioned it in a different way.

STEP 2: introduce 'exchanging' through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

72 - 47

60 70 + 12
- 40 + 7
20 + 5 = **25**

Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract 7, and subtract 4 tens.

STEP 3: Once pupils are secure with the understanding of 'exchanging', they can use the partitioned column method to subtract any 2 and 3-digit numbers.

```
238 - 146 = 92
    100
  200 + 30 + 8
- 100 + 40 + 6
    0 + 90 + 2
```

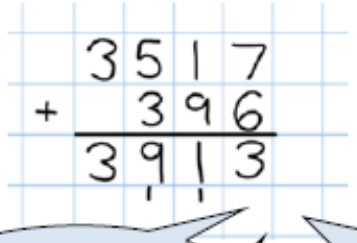
Subtracting money: partition into e.g. £1 + 30p + 8p

Year 4 – Addition and Subtraction

Addition – Add numbers with up to 4 digits.

Move from expanded addition to the compact column method, **adding units first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts.

e.g. 3517 + 396 = 3913



Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition—see Y3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

Add **units** first.

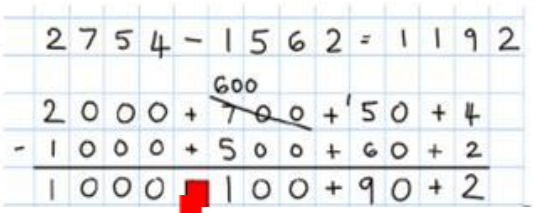
'Carry' numbers **underneath** the bottom line.

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, **not 5 add 3**, for example.

Use and apply this method to money and measurement values.

Subtraction – Subtraction with up to 4 digits.

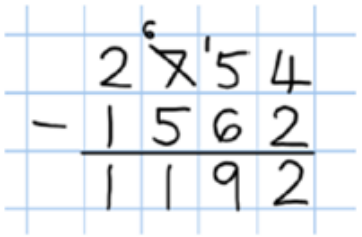
Partitioned column subtraction with 'exchanging' (decomposition):



As introduced in Y3, but moving towards more complex numbers and values. Use **place value counters** to reinforce 'exchanging'.

Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.

Compact column method



To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it (shown on video).

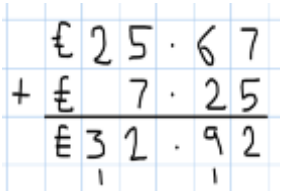
Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved—mental, counting on, counting back or written method (see video).

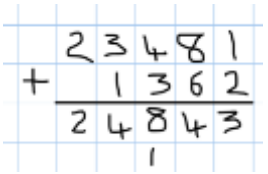
Year 5 – Addition and Subtraction

Addition – Add numbers with more than 4 digits.

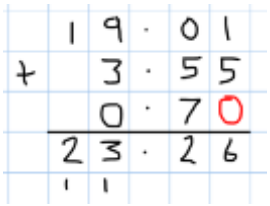
This needs to include money, measures and decimals with different numbers of decimal places.



The decimal point should be aligned in the same way as the other place value columns and must be in the same column as the answer.



Numbers should exceed 4 digits.



Pupils should be able to add more than two values, carefully aligning place value columns.

Empty decimal places can be filled with zero to show the place

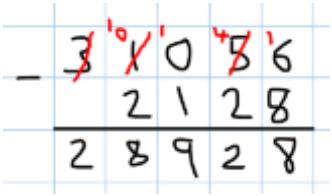
Children should understand the place value of tenths and hundredths and use this to align numbers with different numbers of decimal places.

Subtraction – Subtract with at least 4 digits.

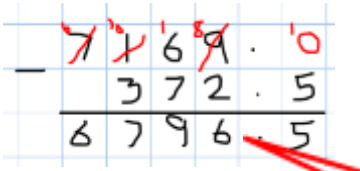
This needs to include money, measures and decimals with different numbers of decimal places.

Compact column subtraction (with exchanging).

Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.



Subtracting with larger integers.



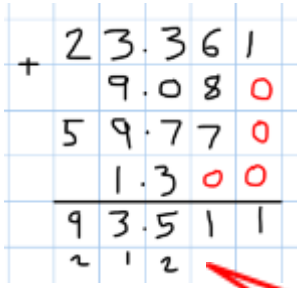
Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Empty decimal places can be filled with zero to show the place.

Create lots of opportunities for subtracting and finding differences with money and measures.

Year 6 – Addition and Subtraction

Addition – Add several numbers of increasing complexity.



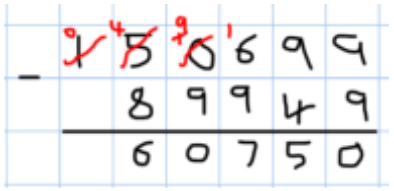
Adding several numbers with different numbers of decimal places, including money and measures.

Decimal points can now be on the line and do not need a separate box of their own.

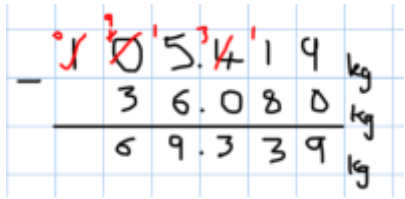
Empty decimal places can be filled with zero to show the place.

Adding several numbers with more than 4 digits.

Subtraction – Subtracting with increasingly larger and more complex numbers and decimal values.



Using the compact method to subtract more complex integers.



Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

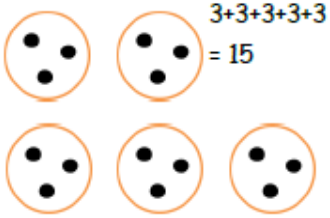
Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting the most appropriate method to work out subtraction problems.

Year 1 – Multiplication and Division

Multiplication – Multiply with concrete objects, arrays and pictorial representations.

How many legs will 3 teddies have?

There are 3 sweets in one bag.
How many sweets are in 5 bags altogether?



- Give children experience of counting equal group of objects in 2s, 5s and 10s.
- Present practical problem solving activities involving counting equal sets or groups, as above.

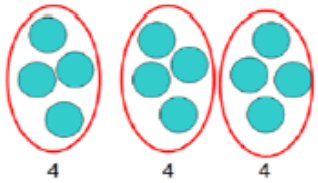
Division – Group and share small quantities.

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing.**

How many groups of 4 can be made with 12 stars? = 3

Grouping:

Sharing:



12 shared between 3 is 4

Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement... ?

"18 shared between 6 people gives you 3 each."

Pupils should :

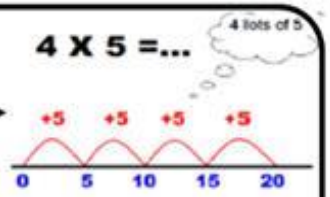
- use lots of practical apparatus, arrays and picture representations
- Be taught to understand the difference between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people)
- Be able to count in multiples of 2s, 5s and 10s.
- Find **half** of a group of objects by sharing into 2 equal groups.

Year 2 – Multiplication and Division

Multiplication – Multiply using arrays and repeated addition (using at least 2s, 5s and 10s).

Use repeated addition on a number line:

- Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.



4 X 5 = 20

Use arrays:

5 x 3 = 15

5 x 3 = 3 + 3 + 3 + 3 = 15

3 x 5 = 5 + 5 + 5 = 15

3 x 5 = 15

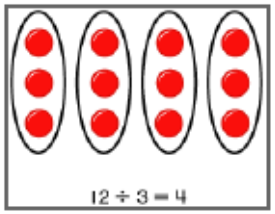
Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as 3 x __ = 6.

Use practical apparatus:

Division – Group and share small amounts using the ÷ and = signs.

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:

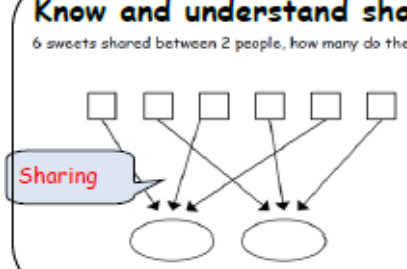


This represents **12 ÷ 3, posed as** how many groups of 3 are in 12?

Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

Know and understand sharing and grouping:

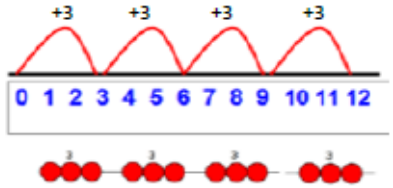
6 sweets shared between 2 people, how many do they each get?



Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out 'how many groups of _ in _ ?'. Pupils could and using a bead string or practical apparatus to work out problems like '*A CD costs £3. How many CDs can I buy with £12?*' **This is an important method to develop understanding of division as grouping.**



12 ÷ 3 = 4

Pose 12 ÷ 3 as 'How many groups of 3 are in 12?'

Year 3 – Multiplication and Division

Multiplication – Multiply 2 digits by a single number digit.

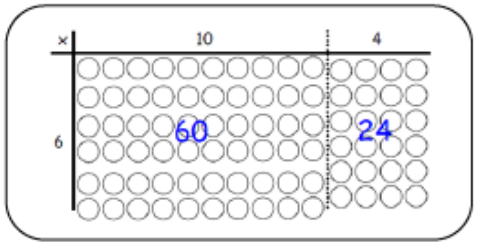
Introduce the grid method for multiplying 2-digit by single-digits:

Eg. 23 x 8 = 184

X	20	3
8	160	24

160 + 24 = 184

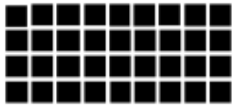
Link the layout of the grid to an array initially:



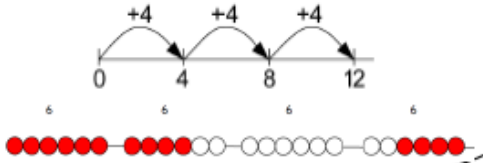
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format (see video clip).

To do this, children must be able to:

- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value
- Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:



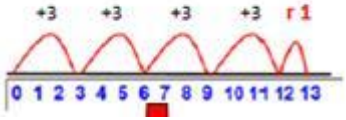
9 x 4 = 36



Division – Divide 2 digit numbers by a single digit.

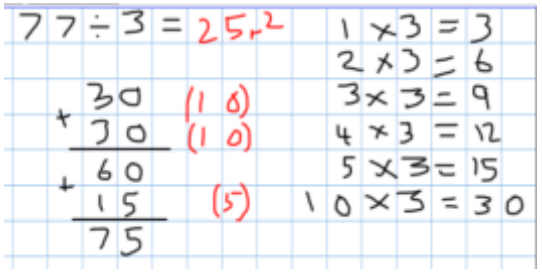
Grouping on a number line:

13 ÷ 3 = 4 r 1



STEP 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of remainders, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for 'carrying' remainders across within the short division method.

Division – using 'chunking up'



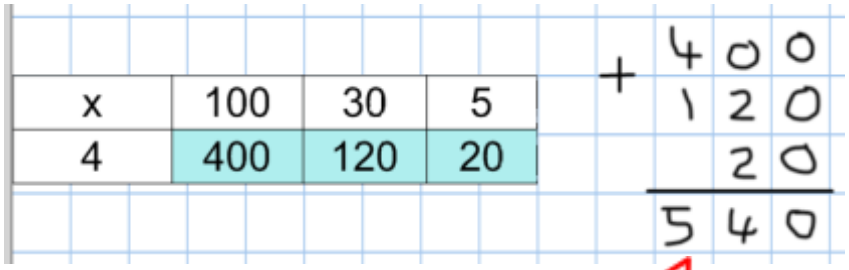
Put the multiple you are adding in brackets. You add these up at the end.

Create a 'partial tables' grid first.

Year 4 – Multiplication and Division

Multiplication – Multiply 2 and 3 digits by a single digit, using all multiplication tables up to 12x12.

Developing the grid method.



x	100	30	5
4	400	120	20

400 + 120 + 20 = 540

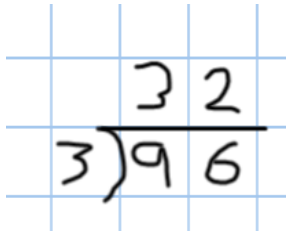
Encourage column addition to add accurately.

Children should be able to:

- Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. Eg. 346 x 9 is approximately 350 x 10 = 3500.
- Multiply multiples of ten and one hundred by a single digit, using their multiplication table knowledge.
- Recall all times tables up to 12 x 12.

Division – Divide 2 and 3 digit numbers by a single digit, with no remainders in the final answer.

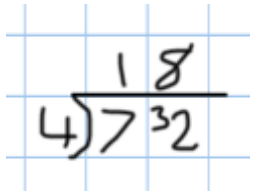
'Bus stop' method for division.



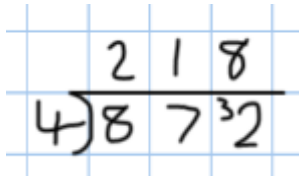
Limited numbers to NO remainders in the answer or carried (each digit must be a multiple of the divisor).

Remind children of the correct place value, that 96 is equal to 90 and 6, but in short division, pose:

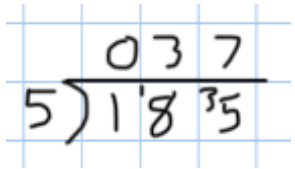
How many 3s in 9? = 3, and record it above the 9 tens.
How many 3s in 6? = 2, and record it above the 6 units.



Limited numbers to NO remainders in the final answer, but with remainders occurring within the calculation process.



Pupils need to move on to dividing numbers with up to 3 digits by a single digit, however problems and calculations provided should not result in a final answer with a remainder at this stage.



When the answer for the first column is zero (1÷ 5 as an example), children could initially write a zero above to acknowledge its place, and must always 'carry' the number (1) over to the next digit as a remainder.

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

Year 5 – Multiplication and Division

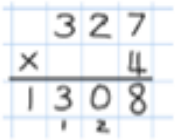
Multiplication – Multiply up to 4 digits by 1 or 2 digits.

Column Multiplication

Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method. Children need to be taught to approximate first, e.g. for 72 x 38, they will use rounding: 72 x 38 is approximately 70 x 40 = 2800, and use this approximation to check the reasonableness of their answer against.

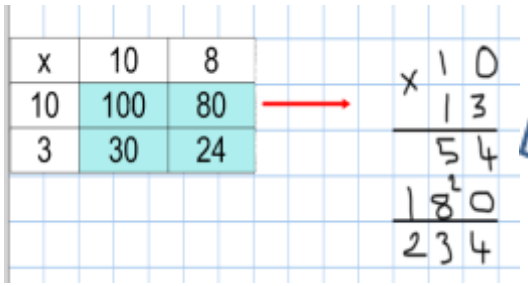
Short multiplication for multiplying by a single digit.

x	300	20	7
4	1200	80	28



Pupils could be asked to work out a given calculation using the grid method, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits.



18 x 3 on the first row.

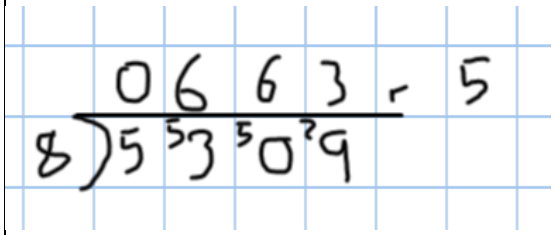
(8x3=24, carrying the 2 for twenty, then '1' x 3)

18x10 on the second row.

Put a zero in the units first, then say 8x1 and 1x1

Division – Divide up to 4 digits by a single digit, including those with remainders.

Short division using the bus-stop method, including remainders in the final answer.



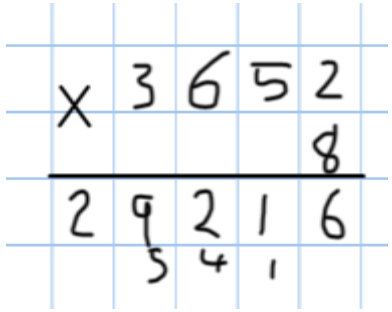
Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it. i.e. as a fraction, decimal, or as a rounded number or value depending upon the context of the problem.

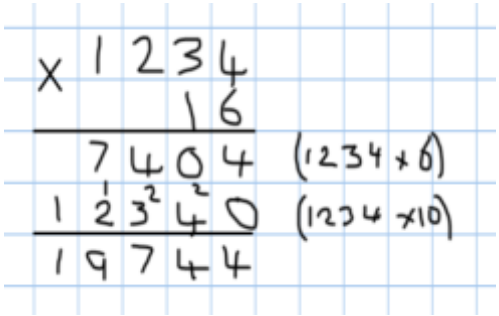
The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, as a decimal or rounded as appropriate to the problem involved.

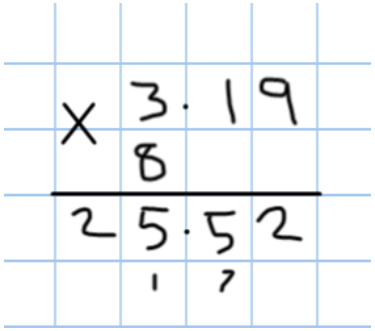
Year 6 – Multiplication and Division

Multiplication – Short and long multiplication, with more complex numbers and multiply decimals with up to 2d.p by a single digit.

Long mutliplication for more complex numbers







This works well for multiplying money (£.p) and other measures.

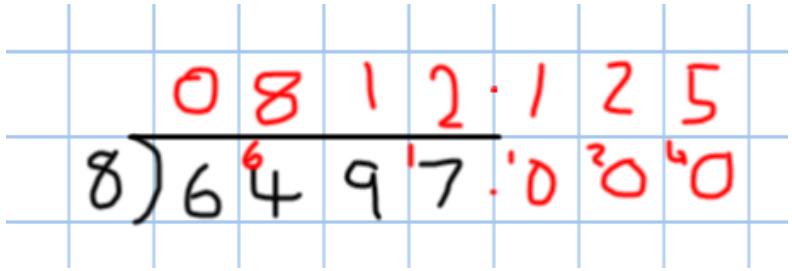
Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.
- Use short multiplication to multiply numbers with more than 4-digits by a single digit; to multiply money and measures, and to multiply decimals with up to 2.d.p. by a single digit.
- Use long multiplication to multiply numbers with at least 4 digits by a 2-digit number.

Division – Divide at least 4 digits by both single digit and 2-digit numbers.

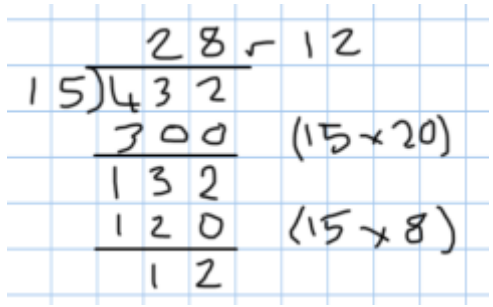
Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as a fraction, decimal, whole number remainders or rounded numbers. Real life problems solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Short division, for dividing by a single digit: e.g. 4697 ÷ 8 =



Calculating a decimal remainder: In this example, rather than expressing the remainder as r. 1 a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

Introduce long division by chunking for dividing by 2 digits



Where remainders occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.